

Assignment 3

Deadline

Friday 17th February, 2017

1. Which of the following statements is/are TRUE for undirected graphs? P: Number of odd degree vertices is even. Q: Sum of degrees of all vertices is even.
2. Let G be a complete undirected graph on 6 vertices. If vertices of G are labeled, then the number of distinct cycles of length 4 in G is equal to.....
3. Simple graphs with $V = n$, Prove that there are $2^{\binom{n}{2}}$ such simple graphs exists.
4. Can we have a graph Q with degree sequence $(1, 2, 2, 3, 3, 5)$, loops and parallel edges allowed?
5. A graph $G = (V, E)$ is called bipartite if V can be partitioned into two sets C and S such that each edge has one vertex in C and one vertex in S . As a specific example, let C be the set of courses at the university and S the set of students. Let $V = C \cup S$ and let $s, c \in E$ if and only if student s is enrolled in course c . (a) Prove that $G = (V, E)$ is a simple graph. (b) Prove that every cycle of G has an even number of edges.
6. What is the maximum number of edges in a bipartite planar graph with n vertices?
7. Can a simple graph have 5 vertices and 12 edges? If so, draw it; if not, explain why it is not possible to have such a graph.
8. How many more edges are there in the complete graph K_7 than in the complete graph K_5 ?
9. For a graph $G = (V, E)$, we define the complement $G' = (V, E')$, where $e \in E'$ if and only if $e \notin E$. Let G be a graph with n vertices, and let A be its adjacency matrix. Write down the adjacency matrix A' of its complement in terms of A and M_n , where M_n is the adjacency matrix of the complete graph K_n .
10. Give an example of a connected graph G where removing any edge of G results in a disconnected graph. Can we determine the number of edges it has?
11. If you construct a graph that captures a relation, describe the properties of a graph representing an equivalence relation.
12. Consider all the possible graphs on n vertices. What can you say about the number of graphs that are connected and those which are disconnected. What fraction of graphs are more in number?
13. We say that G is self-complementary if G is isomorphic to G' . Let G be a cycle on n vertices. Prove that G is self complementary if $n = 5$. Prove that if G is self-complementary, then $n \equiv 0$ or $1 \pmod{4}$.
14. If there are 15 people in a party, is it possible for each of these people to shake hands with exactly three others?
15. Let $G = (V, E)$ be a loop free undirected graph. Prove that if G contains no cycle of odd length then G is bipartite.
16. Let $G(V, E)$ be a loop free undirected n -regular graph with $|V| \leq 2n + 2$. Prove that \overline{G} has a Hamilton cycle.

17. Suppose that someone starts a chain letter. Each person who receives the letter is asked to send it on to four other people. Some people do this, but others do not send any letters. How many people have seen the letter, including the first person, if no one receives more than one letter and if the chain letter ends after there have been 100 people who read it but did not send it out? How many people sent out the letter?
18. The eccentricity of a vertex in an unrooted tree is the length of the longest simple path beginning at this vertex. A vertex is called a center if no vertex in the tree has smaller eccentricity than this vertex. Show that a tree has either one center or two centers that are adjacent.
19. A chessboard has the form of a cross, obtained from a 4×4 chessboard by deleting the corner squares. Can a knight travel around this board, pass through each square exactly once, and end on the same square he starts on?
20. Can 9 line segments be drawn in the plane, each of which intersects exactly 3 others?