Assignment 7

Wednesday 5$^{\text{th}}$ April, 2017

1. Find the generating function for the number of integer solutions to the equation $c_1 + c_2 + c_3 + c_4 = 20$ where $-3 \leq c_1, -3 \leq c_2, -5 \leq c_3 \leq 5, 0 \leq c_4$
2. In how many ways can 2 dozen identical robots be assigned to 4 assembly lines with
   (a) at least 3 robots assigned to each line?
   (b) at least 3, but not more than 9 robots assigned to each line?
3. In how many ways can 3000 identical envelops be divided, in packages of 25, among 4 student groups so that each group gets at least 150, but not more than 1000 of the envelops?
4. In how many ways can we select 7 non-consecutive integers from $\{1, 2, 3 \ldots, 50\}$?
5. In $f(x) = [1/(1-x)][1/(1-x^2)][1/(1-x^3)]$, the co-efficient of x^6 is 7. Interpret this result in terms of partitions of 6.
6. Show that the number of partitions of a positive integer n where no summand appears more than twice equals the number of partitions of n where no summand is divisible by 3.
7. Show that the number of partitions of $n \in \mathbb{Z}^+$, where no summand is divisible by 4 = the number of partitions of n where no even summand is repeated (although odd summands may or may not be repeated).
8. Using a Ferrers graph, show that the number of partitions of an integer n into summands not exceeding n is equal to the number of partitions of n into at most m summands.
9. Find the exponential generating function for the sequence $0!, 1!, 2!, 3!, \ldots$