

Assignment 6: Principle of Inclusion and Exclusion

Monday 3rd April, 2017

1. At Rekha's muffin shop Rekha wants to arrange 15 different muffins on 5 shelves for a window display. In how many ways can she arrange them so that each shelf has at least 1, but no more than 4 muffins.
2. If 8 distinct dice are rolled what is the probability that all 6 numbers appear.
3. At a 12 week school in Cryptography Varsha met 7 of her friends from college, during the conference, she met each of them at lunch 35 times, every 'couple' of them 16 times, every 'trio' 8 times, every 'quad' 4 times, each 'set of five' 2 times and each 'set of six' once. Never did Varsha see all 7 at once. If she had lunch everyday during the 84 days of the school, did she ever have lunch alone.
4. In how many ways can one arrange the letters in CORRESPONDENTS so that
 - (a) there is no pair of consecutive identical letters?
 - (b) there are exactly two pairs of consecutive identical letters?
 - (c) there are at least three pairs of consecutive identical letters?
5. Let $A = \{1, 2, 3, \dots, 10\}$ and $B = \{1, 2, 3, \dots, 7\}$. How many functions $f : A \rightarrow B$ satisfy $|f(A)| = 4$? How many have $|f(A)| \leq 4$?
6. If 13 cards are dealt from a standard deck of 52, what is the probability that these 13 cards include
 - (a) at least one card from each suit?
 - (b) exactly one void (for example no clubs)?
 - (c) exactly two voids?
7. Ten goons attend a business luncheon. Each goon at the reception submits his coat and gun. Upon leaving, each goon is given a coat and a gun at random.
 - (a) In how many ways can the coats and the guns be distributed so that no goon gets either of his possessions?
 - (b) In how many ways can they be distributed so that no goon gets back both of his possessions?
8. Mr. Galton teaches geometry and then biology to a class of 12 advanced students in a classroom that has only 12 desks. In how many ways can he assign the students to these desks so that
 - (a) no student is seated at the same desk for both the classes?
 - (b) there are exactly 6 students each of whom occupies the same desk for both the classes?
9. Prof. Vishwamitra has 5 TAs to correct programs in his courses in Java, C++, SQL, Pearl and Verilog. TA Ramya and Soumya both dislike SQL, Sunayana wants to avoid C++ and Verilog, Pooja detests Java and C++, Raj refuses to work in SQL and Pearl. In how many ways can Prof. Vishwamitra assign each TA to correct programs in one language, cover all 5 languages and keep everyone happy.

10. A pair of dice, one red and other other green, are rolled 6 times. We know that the ordered pairs $(1, 1)$, $(1, 5)$, $(2, 4)$, $(3, 6)$, $(4, 2)$, $(4, 4)$, $(5, 1)$ and $(5, 5)$ did not come up. What is the probability that every value came up on both the red die and the green die?
11. For $A = \{1, 2, 3, 4, 5\}$ and $B = \{u, v, w, x, y, z\}$, determine the number of one to one functions $f : A \rightarrow B$ where $f(1) \neq v, w$; $f(2) \neq u, w$; $f(3) \neq x$; and $f(4) \neq v, x, y$.