CSL105 : Discrete Mathematics

Major Examination

Indian Institute of Technology Ropar

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Total Duration : 3 hours | Total Marks : 100 M

Section I [5 Marks each]

1. What are the total number of passwords with at least 6 digits and at most 8 digits, with a condition that there must at least be one capital letter and one numeral. Prove your answer.
2. Show that for every bijective function f, there exists an inverse.
3. Establish the validity of the following and provide reasons :

$$
\begin{array}{r}
\neg p \vee s \\
\neg t \vee (s \wedge r) \\
\neg q \vee r \\
p \vee q \vee t \\
---- \\
r \vee s \\

\end{array}
$$

4. State and prove the *principle of inclusion and exclusion* through induction on t:

$$\overline{N} = N - \sum_{1 \leq i \leq t} N(c_i) + \sum_{1 \leq i \leq j \leq t} N(c_i c_j) - \sum_{1 \leq i \leq j \leq k \leq t} N(c_i c_j c_k) + \cdots + (-1)^t N(c_1 c_2 c_3 \ldots {}_t)$$

5. A tree has a Hamilton Path iff _________. Prove your answer.
6. Derive the chromatic polynomial of a cycle on 5 vertices?
7. How do you check the divergence of an infinite sequence using quantifiers? Explain.
8. Six married couples are to be seated at a circular table. In how many ways can they arrange themselves so that no wife sits next to her husband.
9. Show that isomorphism of graphs is an equivalence relation.
10. Find the value of the following. Provide a story proof used in determining the values:

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots \binom{n}{n} \tag{1}$$

$$\binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \ldots \binom{n}{n}^2 \tag{2}$$

Section II [10 Marks each]

1. Show that every connected graph with n vertices has at least $n-1$ edges. Give a rigorous proof.

2. In how many ways can you triangulate a regular polygon having $n+2$ sides?

3. A pair of dice, one red and other other green, are rolled 6 times. We know that the ordered pairs (1, 1), (1, 5), (2, 4), (3, 6), (4, 2), (4, 4), (5, 1) and (5, 5) did not come up. What is the probability that every value came up on both the red die and the green die?

4. One version of Ackermann's function $A(m,n)$ is defined recursively for $m,n \in \mathbb{N}$ by :
   $A(0,n) = n+1,\ \ n \geq 0$
   $A(m,0) = A(m-1,1), m > 0\ and$
   $A(m,n) = A(m-1, A(m,n-1)),\ \ m,n > 0$
   Such functions were defined in the 1920s by the German mathematician and logician Wilhelm Ackermann and play an important role in computer science - in the theory of recursive functions and in the analysis of algorithms that involve the union of sets.
   Verify that $A(3,n) = 2^{n+3} - 3\ for\ all\ n \in \mathbb{N}$.

5. How many 20-digit quaternary (0,1,2,3) sequences are there where there is at least one 2 and an odd number of 0s.