## CSL105 : Discrete Mathematics Minor Examination Indian Institute of Technology Ropar Instructor: Dr. Sudarshan Iyengar

## March 2017

Total Duration : 2 hours

Total Marks : 80 M

Section I	[5 Marks eac	h]

- 1. Let  $S = \{1, 2, 3, ...\}$ . Consider a relation  $R = \{(a, b)/a + b \le 10\}$ . Prove or disprove that R satisfies reflexive, symmetric, antisymmetric and transitive properties.
- 2. In the matrix representation of a relation, how does one find if the relation is transitive or not? Prove that your method works.
- 3. Every sequence of  $n^2 + 1$  distinct real numbers contains a subsequence of length n + 1 that is either strictly increasing or strictly decreasing.
- 4. State well ordering principle. State and Prove Mathematical Induction.
- 5. What is the condition for a function to be invertible? Explain with an example.
- 6. Six boxes are colored red, black, blue, yellow, orange and green. In how man ways can you put 20 identical balls into these boxes such that no box is empty?
- 7. If R is a reflexive relation on S, then so is any superset of R inside S S.
- 8. Let G = (V, E) be a loop free undirected graph. Prove that if G contains no cycle of odd length than G is bipartite.

## Section II

## [10 Marks each]

1. Prove by Induction that  $1 + \frac{1}{2} + \frac{1}{3} + \ldots = \infty$ .

2. Show that

$$1.2.3 + 2.3.4 + 3.4.5 + \ldots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

- 3. You need to choose a password which is at least 6 characters and at most 8 characters in length with an added condition that each character is an uppercase letter or a digit. Also, your password must contain at least one digit. In how many ways can you choose your password?
- 4. Enumerate all possible non-isomorphic graphs on 4 vertices.