# CSL105 : Discrete Mathematics <br> Minor Examination <br> Indian Institute of Technology Ropar <br> Instructor: Dr. Sudarshan Iyengar 

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## Section I

[5 Marks each]

1. Let $S=\{1,2,3, \ldots\}$. Consider a relation $R=\{(a, b) / a+b \leq 10\}$. Prove or disprove that $R$ satisfies reflexive, symmetric, antisymmetric and transitive properties.
2. In the matrix representation of a relation, how does one find if the relation is transitive or not? Prove that your method works.
3. Every sequence of $n^{2}+1$ distinct real numbers contains a subsequence of length $n+1$ that is either strictly increasing or strictly decreasing.
4. State well ordering principle. State and Prove Mathematical Induction.
5. What is the condition for a function to be invertible? Explain with an example.
6. Six boxes are colored red, black, blue, yellow, orange and green. In how man ways can you put 20 identical balls into these boxes such that no box is empty?
7. If $R$ is a reflexive relation on $S$, then so is any superset of $R$ inside $S$.
8. Let $G=(V, E)$ be a loop free undirected graph. Prove that if G contains no cycle of odd length than G is bipartite.

## Section II

1. Prove by Induction that $1+\frac{1}{2}+\frac{1}{3}+\ldots=\infty$.
2. Show that

$$
1.2 .3+2.3 .4+3.4 .5+\ldots+n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}
$$

3. You need to choose a password which is at least 6 characters and at most 8 characters in length with an added condition that each character is an uppercase letter or a digit. Also, your password must contain at least one digit. In how many ways can you choose your password?
4. Enumerate all possible non-isomorphic graphs on 4 vertices.
