

# First test CSL105

3rd February 2017

1. There are  $n$  people present in a room. Prove that among them there are two people who have the same number of acquaintances in the room. (1 mark)
2. In 2016, there were 35,000 rank holders in some national level entrance exam, with same rank being shared by multiple people. Alice, Bob, Cathy, Dirac and Elisa cleared this exam. In how many ways can these 5 students secure their ranks? (2 marks)
3.  $S(8, 5) = ?$ . (2 marks)
4. Give an example of a function  $f : A \rightarrow B$  and  $A_1, A_2 \subseteq A$  for which  $f(A_1 \cap A_2) \neq f(A_1) \cap f(A_2)$ . (2 marks)
5. State and prove the generalized pigeon hole principle. (2 mark)
6. During a month with 30 days, a baseball team plays at least one game a day, but no more than 45 games. Show that there must be a period of some number of consecutive days during which the team must play exactly 14 games. (2 marks)
7. An odd number of people stand in a park at mutually distinct distances. At the same time each person throws a stone at their nearest neighbor, hitting this person. Use mathematical induction to show that there is at least one survivor, that is, at least one person who is not hit by a stone. (4 marks)