Tutorial 1

Friday 6$^{\text{th}}$ January, 2017

1 Counting Problems

1. What is the value of k after the following code snippet is executed :

   (a)
   ```
   k=0
   for i₁ = n₁
       for i₂ = n₂
           for i₃ = n₃
               ⋮
               for iₘ = nₘ
                   k=k+1
   ```

   (b)
   ```
   k=0
   for i₁ = n₁
       k=k+1
       for i₂ = n₂
           k=k+1
           for i₃ = n₃
               k=k+1
               ⋮
               for iₘ = nₘ
                   k=k+1
   ```

2. In how many ways can 10 men and 10 women be seated in a row if:

   (a) Any person can sit next to any other.

   (b) Men and women occupy alternate seat.

   (c) Husband and wife sit together.

3. A committee of 8 has to be chosen out of 16 men and 10 women. In how many ways can this be done if:

   (a) No restrictions.

   (b) The committee must include equal men and women.

   (c) The committee must include 7 women

   (d) The committee must include more women than men.

   (e) The committee must include at least 6 men.

4. Compute the value of the following:

   (a) $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n}$

   (b) $\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \cdots + (-1)^n \binom{n}{n}$

2 Permutations with Repetitions

1. Amit, Nihal, Shrikanth, Rohan, Neelam and Rashmi have a Giani's free ice-cream coupon each, in which they can avail the following flavors : Chocolate, Vanilla, Strawberry, Butterscotch. In how many ways can they buy ice-creams? Is it the same as the number of ways in which the vendor at Giani can sell them 6 ice-creams? Justify your answer.

2. There are three bins, each containing red, green, and blue balls respectively. How many arrangements of 5 balls can be made if each bin has unlimited supply of balls?

3. In how many ways can 5 people A, B, C, D, E be arranged on a circular table, such that:

   (a) A and B are always seated together.
   (b) C and D never sit together.

4. What is the number of subsets of a set with n elements?

5. What is the number of solutions to:

$$x_1 + x_2 + x_3 + x_4 = 7, \text{where} x_i \geq 0 \text{and} \forall i 1 \leq i \leq 4 \tag{1}$$

6. Count the number of ways in which 3 men and 3 women can be seated in a round table such that no two men sit together.

7. What would the coefficient of:

   (a) $x^5 y^2$ be in the expansion of $(x + y)^7$?
   (b) $a^5 b^2$ be in the expansion of $(2a - 3b)^7$?