Tutorial 4 - Functions and Relations

24th & 25th January 2017

1. State the pigeon hole principle.
2. Show that given a sequence of $n^2 + 1$ positive integers, one can always find a subsequence of length $n + 1$ in increasing or decreasing order. Note the usage of the pigeon hole principle.
3. Show that loseless compression is theoretically impossible to achieve.
4. If 54 students who are crediting this course, stand up and start shaking hands with others, randomly, is there a possibility that no two people shake hands the same number of times?
5. Show that given an odd integer m, there always exists an integer n such that $m|2^n - 1$.
6. Let $S = \{1, 2, 3, \ldots, 200\}$, pick 101 distinct elements randomly from this set, say T=$a_1, a_2, \ldots, a_101$. Show that one can always find two elements in T such that one divides the other.
7. Given that five points lie on a sphere, prove that there exists a closed semi-sphere, which contains 4 of the points. Note that semi-sphere includes the boundary.
8. Given 5 points in a square with side 1 cm, show that one can always find two points who are less than or equal to $2sqrt(2)$ cm apart.
9. Show that there are at least two women men in Ropar who have the same amount of hair strands :-).
10. State and prove the extended pigeon hole principle.
11. Show that given 6 people, you can always find 3 people who know each other or 3 people who don't know each other.
12. A lady checks at least 1 and at most 132 matrimonial profiles per day for the next 11 weeks. Show that there is a sequence of successive k days, for some k, where she has dated 21 people.